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A digital approach to integration of accelerometer data

S.S. Law[I]

ABSTRACT

The direct integration of acceleration record introduces offset and slope in the resulting displacement record. Several different balancing or baseline correction procedures have been proposed over the years. All of these approaches require the knowledge of the initial or final velocity and displacement. A direct approach to the integration of accelerometer data is presented which does not depend on the values of velocity and displacement both at the beginning and end of the event. A digital filtering procedure is developed and checked against simulated data and laboratory results. High accuracy is obtained with the proposed method.

INTRODUCTION

Earthquake acceleration record represents a random physical phenomenon which may be considered as a combination of multiple sine waves of different magnitude at different frequencies. In the Fast Fourier Transform of the time series, each sine wave is represented by a spectral peak in the frequency spectrum of the signal. The offset to the mean and slope of a times series are respectively the DC component and low frequency components in the power spectrum.

Integration of acceleration data introduces offset and slope in the resulting displacement record. Fig. 1 shows that for a signal with a given slope, the slope is represented by low frequency components up to 1 Hz in the frequency spectrum for a sampling rate of 512 samples/second. The magnitude of the DC component is proportional to the value of slope. Several different balancing or baseline correction procedures have been proposed over the years

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[I] Senior Lecturer, Civil & Structural Engg. Dept., Hong Kong Polytechnic

(Trifunac 1970; Pecknold and Riddell 1978). Dynamic programming approach (Trujillo and Carter 1982; Trujillo 1978) was also proposed to solve the problem. All of these approaches require the knowledge of the initial or final velocity and displacement. The characteristics of the procedure which is of interest here is that it does not require any knowledge on the initial or final velocity and displacement of the event. The aim of the procedure described below is to minimize the DC and near DC components of the signal incurred during the process of integration.

FINITE IMPULSE RESPONSE (FIR) FILTER

The primary purpose of digital filtering is to alter the spectral information contained in an input signal. While this can be accomplished in either the time or frequency domain, FIR is used here in the time domain aspect of filtering. One of the characteristics of FIR is its linear phase response. An input signal goes through the filter would has its phase characteristic retained after filtering. This makes it most suitable for the present work.

THE PROCEDURE

Earthquake acceleration record usually lasts for several minutes and it is always possible to have a digitized time series to cover the whole event. It is assumed that a good knowledge of the zero datum of acceleration is available. Trapezoidal rule is used in the integration.

Step 1: Remove the offset in the acceleration record and perform first integration

2. The velocity record is fitted with a regressed straight line for the values of offset and slope which are then subtracted from the velocity record.

3. Perform second integration.

4. The power spectrum of acceleration record and displacement record obtained from Step 3 are compared to identify the frequency range at which error has been introduced.

5. The displacement record is then highpass filtered with a Finite Impulse Response (FIR) Filter at the cutoff frequency determined from Step 4.

SIMULATED DATA

A series of data is generated at 512 samples/second as a combination of sine wave at 10 Hz and an offset of 3.0 plus a slope of 1 percent. By following the procedure described above, the effect of offset and slope in the integration process is removed and Fig. 2 compares the original signal with the signal after integrating twice with appropriate scale factor included. Since the original signal is a sine wave, it is shown as the mirror image about the zero datum for comparison. The two signals are almost the same except for some distortion at the beginning and end of the filtered signal which are due to the FIR filtering.

EXPERIMENTAL DATA

The procedure is further tested by comparing the displacement obtained by integration and displacement obtained from displacement trandsucer. Two tests were carried out on a suspended structure in the laboratory. The experimental setup is shown in Fig. 3. Signal from accelerometer model B&K 8306 and linear voltage displacement tranducer (LVDT) model TML CDP-50 at a point of the structure were simultaneously lowpassed and digitized at 1024 samples per second. The analogue/digital conversion subroutine is such written that the time difference between adjacent channel so sampled is approximately 0.0001 second. There is virtually no phase difference between data in adjacent channel. 2048 weights Hanning's window was used in the subsequent highpass filtering of data. The transition bandwidth between passband and stopband is only 2 Hz for overshoot of less than 0.3% in the achieved filter response.

The input excitation are separately 20 Hz sinusoidal signal, and 31.6 Hz bandwidth narrow band random noise centred at 20 Hz. Figs. 4 to 7 show the comparison of the frequency responses from different sensors after integration. There are strong low frequency components introduced by integration for both types of input. Comparing with Figs. 8 and 9 shows that the magnitude of displacement spectral peaks at frequencies greater than 3 Hz are almost the same for the two sensors. This indicates that all the frequency components in the acceleration record are retained in the integration. The comparison of displacement time series in Figs. 10 and 11 show that in the case of sinusoidal input, the responses are very similar whereas they are not alike in the case of random input. There are also slight distortion at the beginning and end of the filtered signal.

ACCURACY

Table 1 shows details of the procedure on all the tests with the accuracy calculated basing on the root mean square (RMS) value of the data. It is known that FIR filter introduces distortion at the beginning and end of filtered data as seen in the experiment. Hence the first 300 samples and last 100 samples in the filtered signal are not considered in the calculation of RMS values.

The error in the simulated data is only +3.45 %. In the case of laboratory test, the RMS error is again low at +3.10 % for sinusoidal input and +8.29 % in the case for random input. This error is mainly due to the attenuation of the original signal near the cutoff frequency. Since the effect of slope in the spectrum is within DC to 0.1 Hz for 64 samples/second sampling rate as compared to DC to 1 Hz for 512 samples/second, this polluted region is isolated from the region containing significant information in the usual earthquake acceleration record. The exact cutoff frequency can be predicted by comparing the power spectrum of acceleration and integrated displacement record, and the undesirable low frequency components can be accurately eliminated with a more sharp cutoff transition by inclusion of a still larger number of filter weights in the highpass filter.

For earthquake record containing the major information within the frequency range from DC to 5 Hz, say, a digitizing rate of 64 samples/second would exhibit the relationship as shown in Table 2 between the transition bandwidth and the number of filter weights. This would still maintain an overshoot of less than 0.3% in the achieved filter response.

CONCLUSION

A simple digital approach has been tested on both sinusoidal and random acceleration records and accurate displacement history has been obtained after suitable digital filtering with a RMS error of 3% to 8%. The use of a larger number of weights in the highpass filter and appropriate sampling rate could reduce the upper limit of error to 5%.

REFERENCE

Pecknold D.A. and Riddell R. 1978. Effect of Initial Base Motion on Response Spectra. J. Engg. Mechanics, ASCE, 104(4).

Trifunac M.D. 1970. Low Frequency Digitization Errors and a New Method of Zero Baseline Correction of Strong-Motion Accelerograms. Earthquake Engg. Research Laboratory, EERL 70-07, California I. of Technology, Pasadena, Calif.

Trujillo D.M. 1978. Application of Dynamic Programming to the General Inverse Problem. Int. J. for Numerical Methods in Engg., Vol.12.

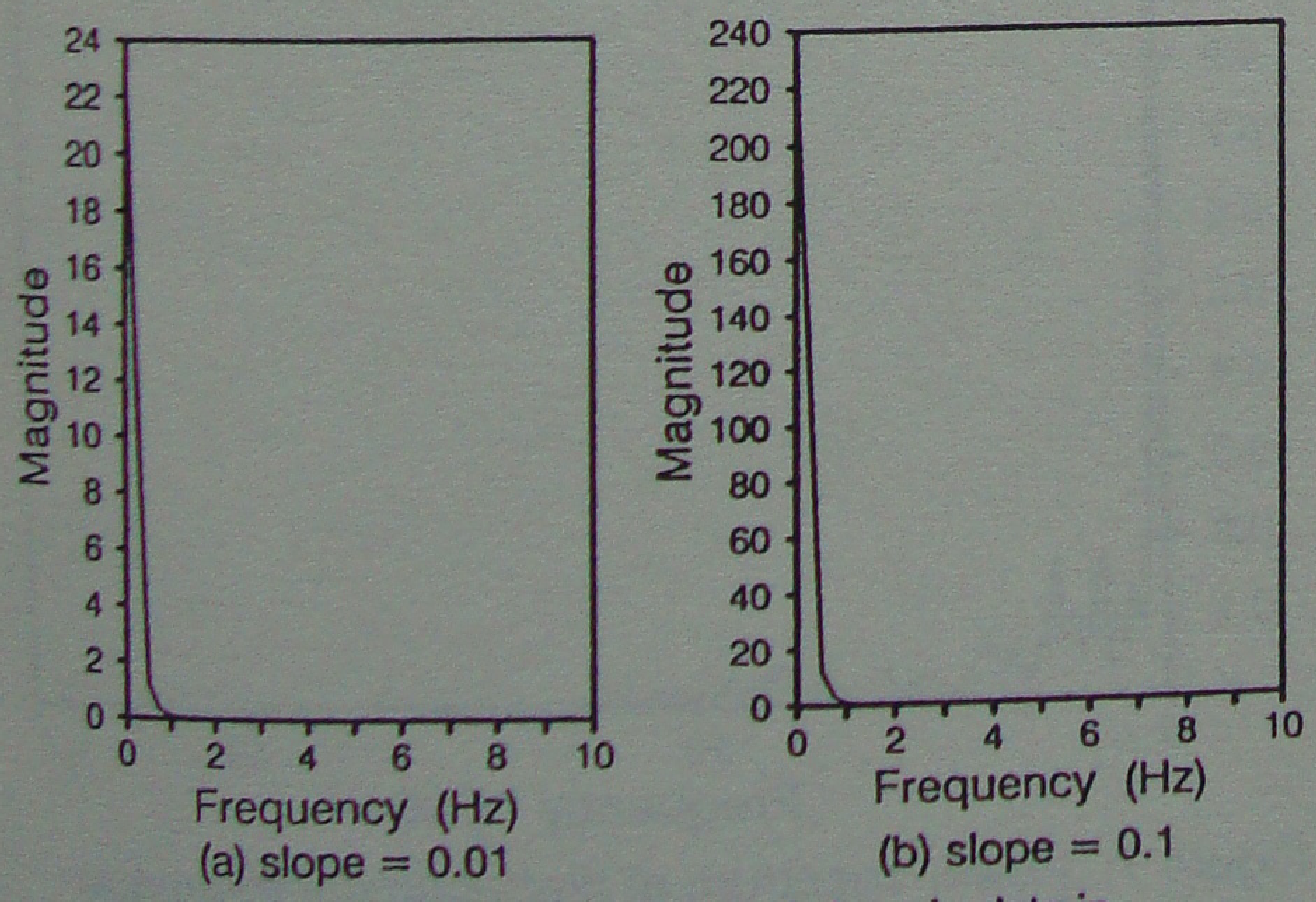
Trujillo D.M. and Carter A.L. 1982. A New Approach to the Integration of Accelerometer Data. Earthquake Engg. and Structural Dynamics, Vol.10.

Table 1

Data Type			No. of data	Highpass filter at	RMS value	% error in RMS
Simulated	Original	Sinusoidal plus noise	2048	-	2.8214	
	After integration		2048	6.0 Hz	2.9222	+3.45
Displacement	(LVDT)	Sinusoidal	4096	-	0.11699	
Acceleration	After integration	Sinusoidal	4096	10.0 Hz	0.12074	+3.10
Displacement	(LVDT)	Random	4096	-	0.22067	
Acceleration	After integration	Random	4096	2.0 HZ	0.24062	+8.29

Table 2

Transition Bandwidth	Filter weights
0.5 Hz	512
0.25 Hz	1024
0.125 Hz	2048



(a) slope = 0.01

(b) slope = 0.1

Figure 1: Representation of slope in data in power spectrum

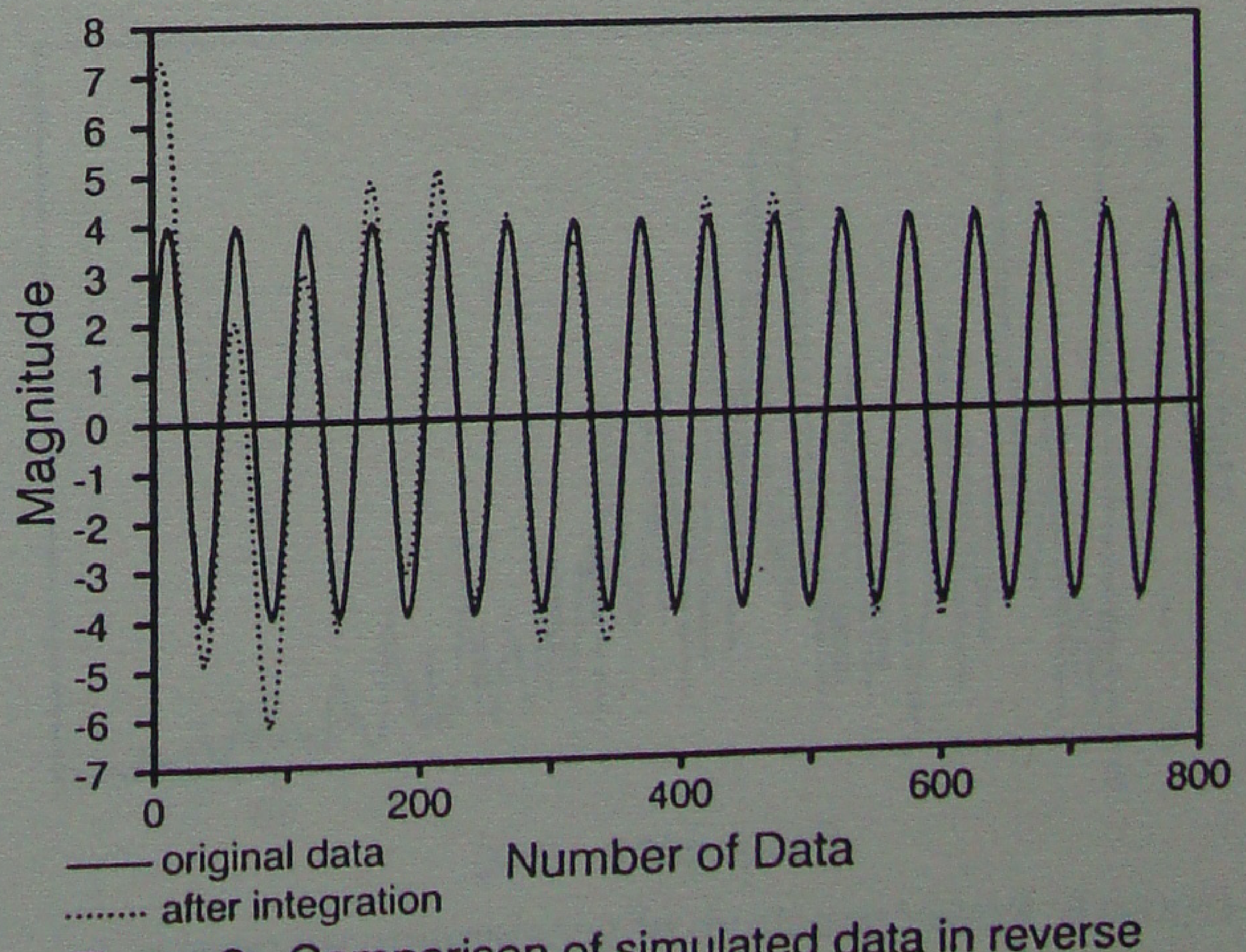


Figure 2: Comparison of simulated data in reverse and data after integration

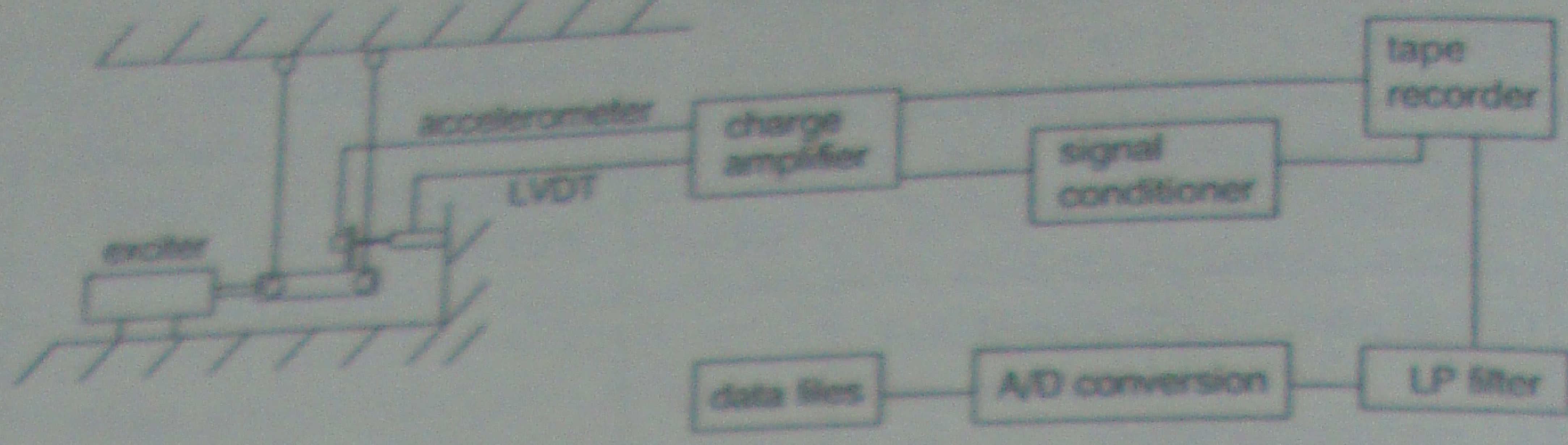


Figure 3: Arrangement of equipment in laboratory test

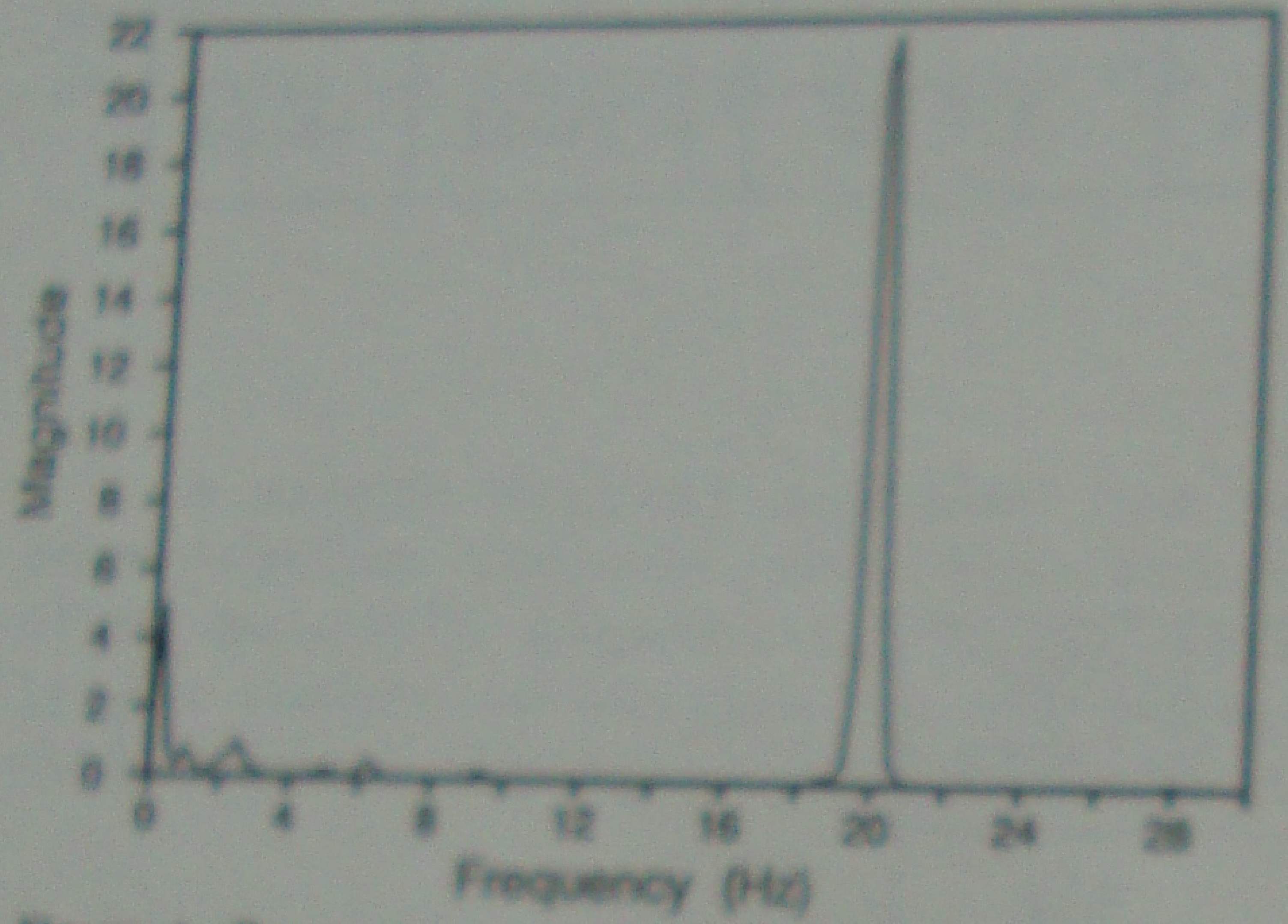


Figure 4: Power spectrum of acceleration response from accelerometer, 20 Hz sine wave input

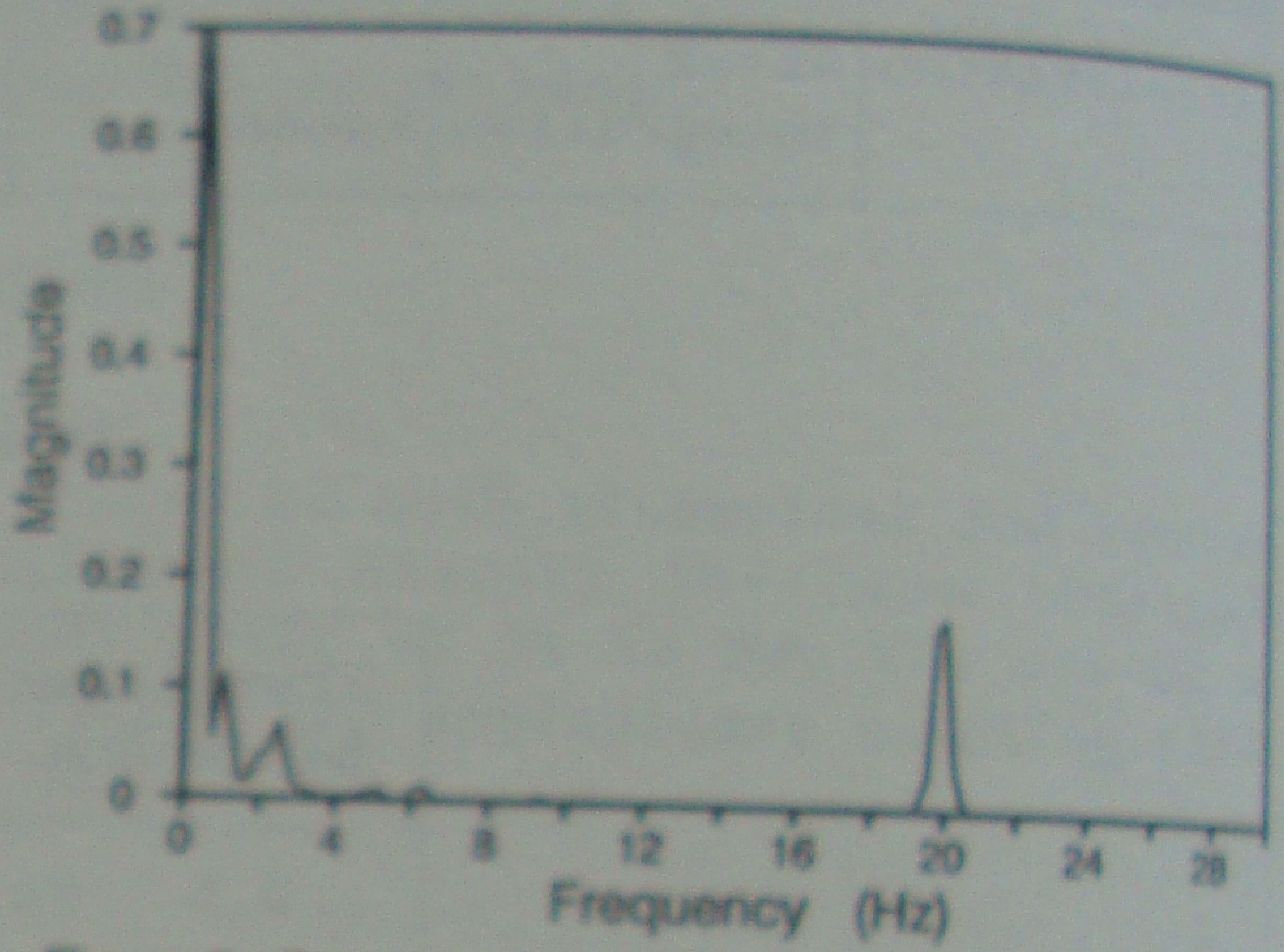


Figure 5: Power spectrum of displacement response from acceleration record after integration 20 Hz sine wave input

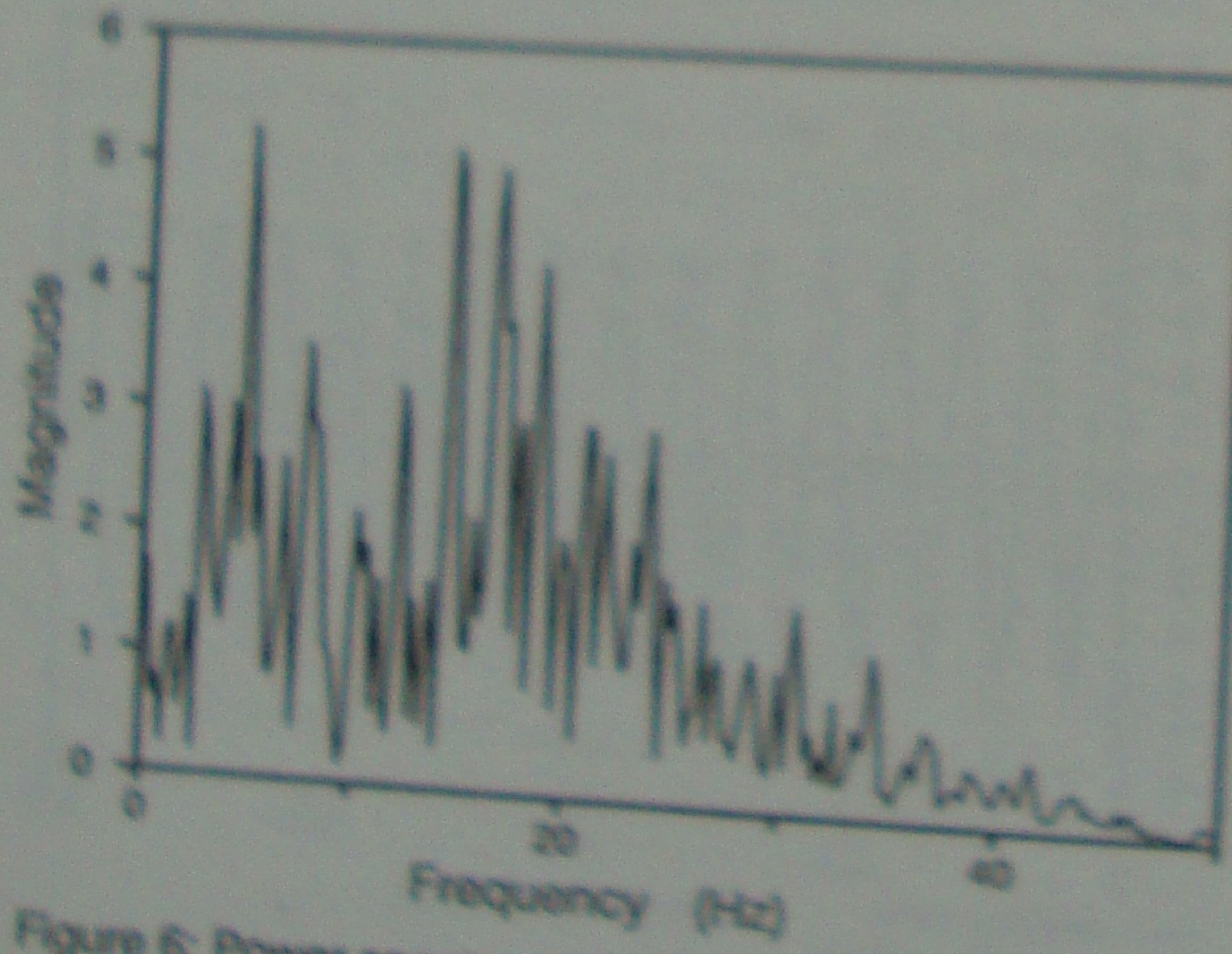


Figure 6: Power spectrum of acceleration response from accelerometer, random input

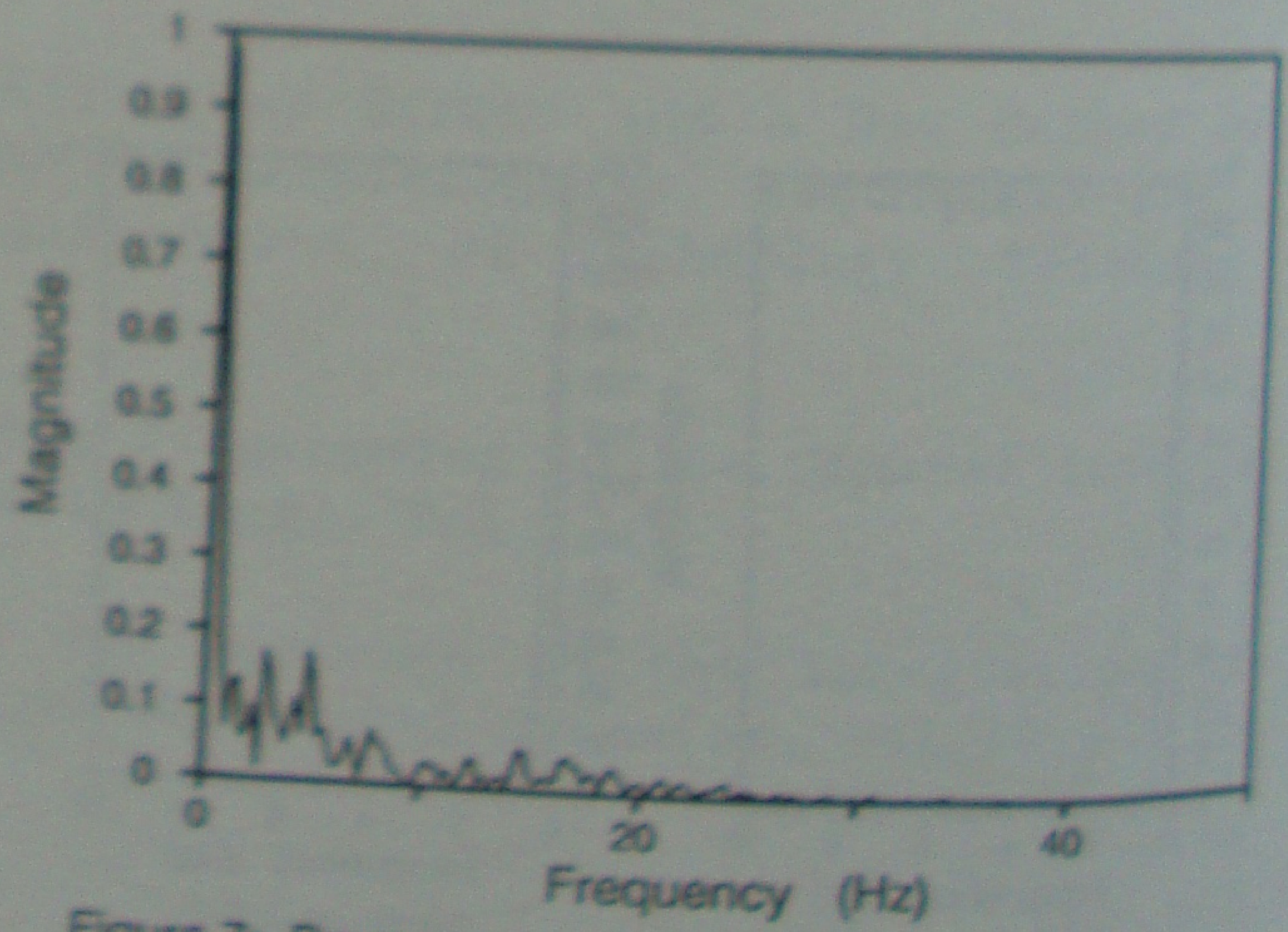


Figure 7: Power spectrum of displacement response from acceleration record after integration random input

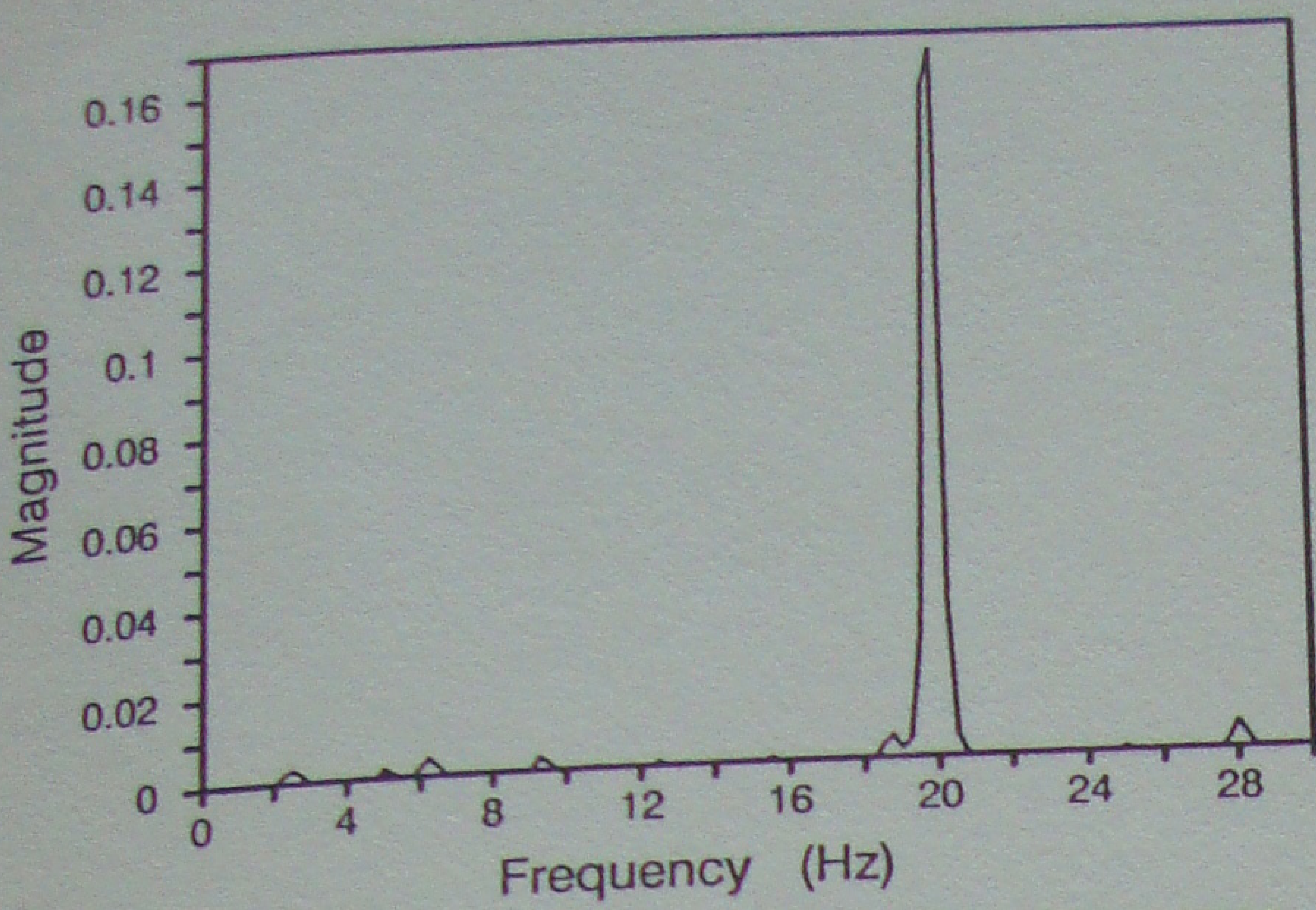


Figure 8: Power spectrum of displacement response from LVDT, 20 Hz sine wave input

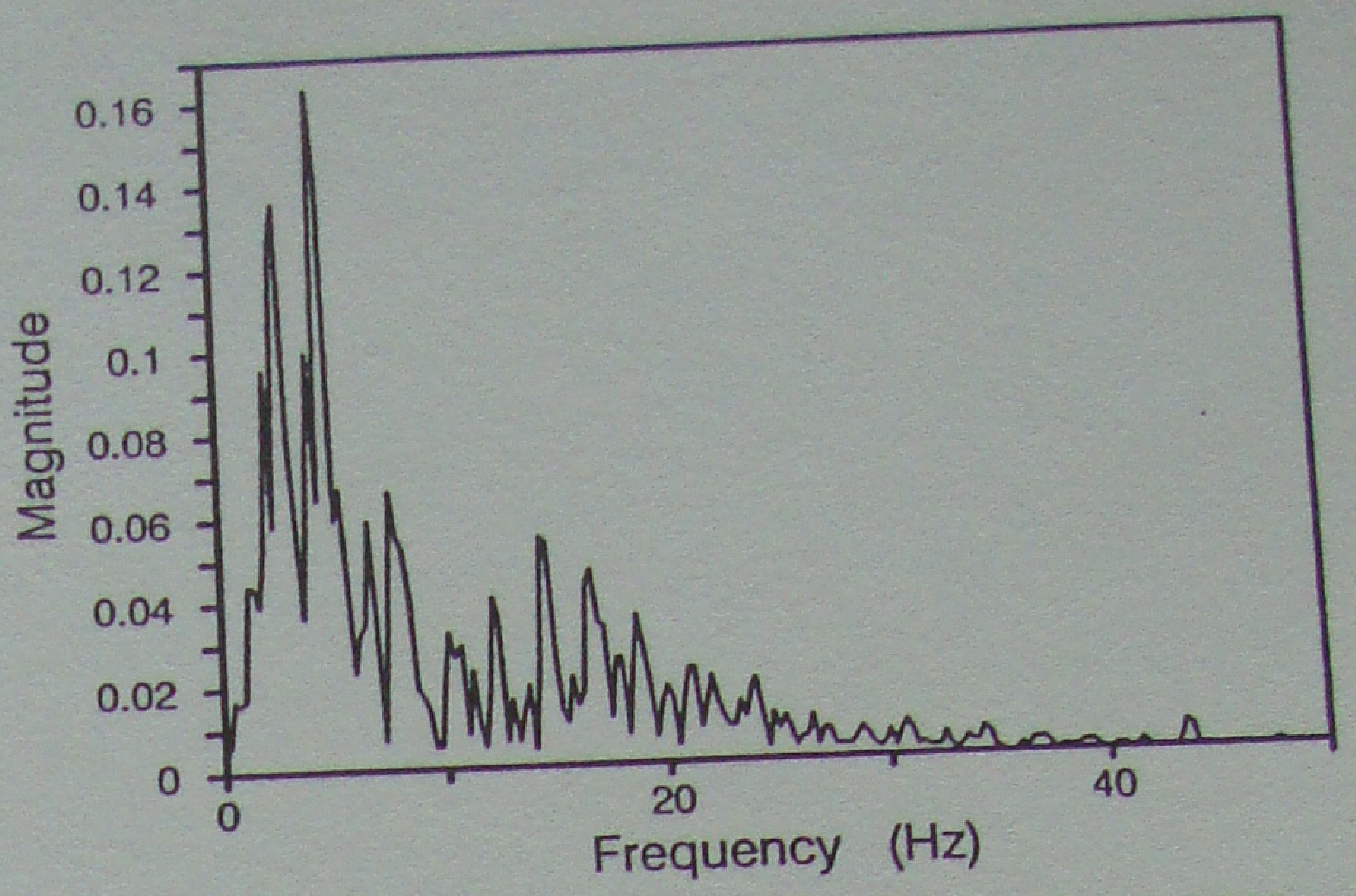


Figure 9: Power spectrum of displacement response from LVDT, random input

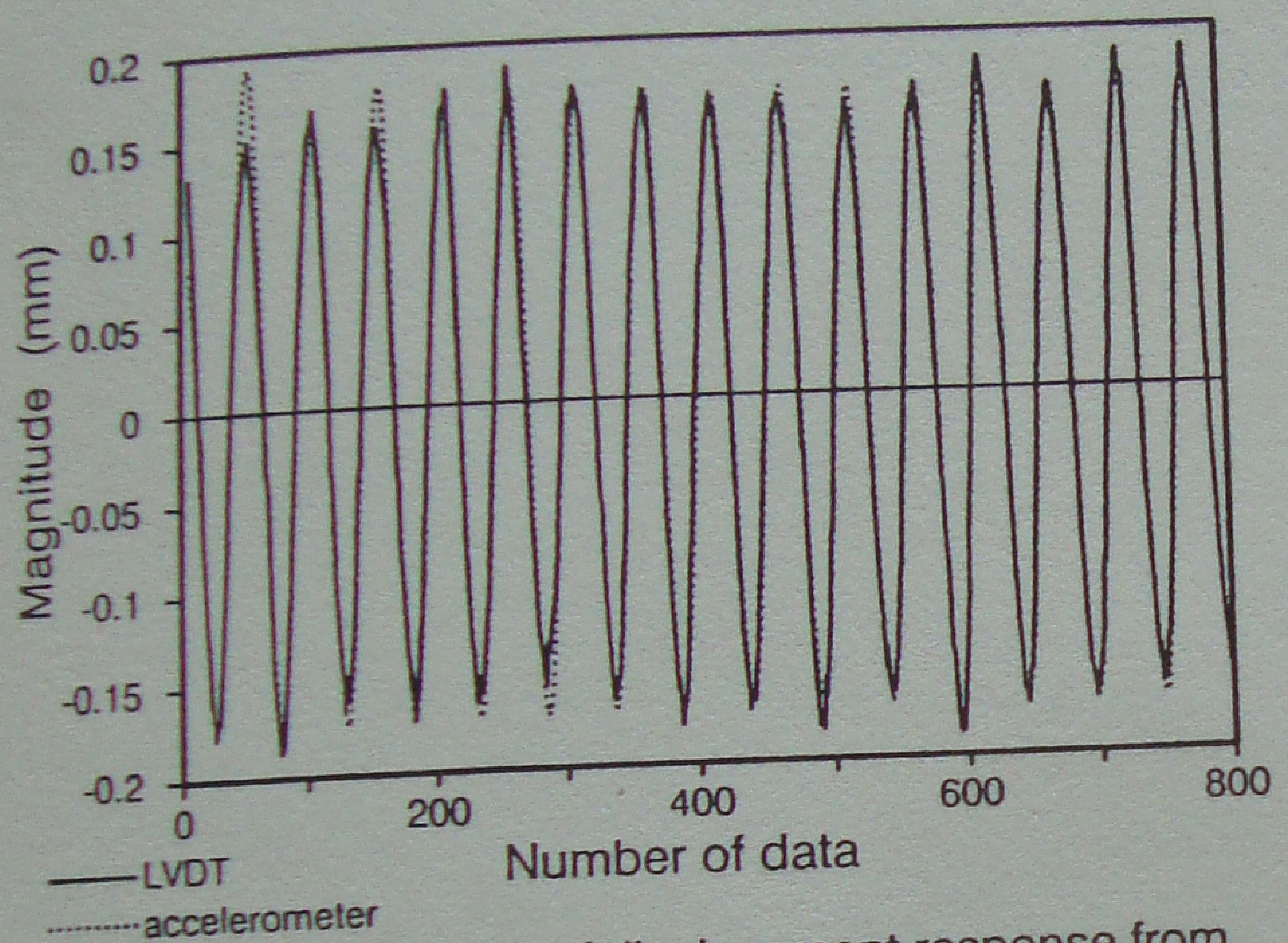


Figure 10: Comparison of displacement response from LVDT and accelerometer, 20 Hz sine wave input

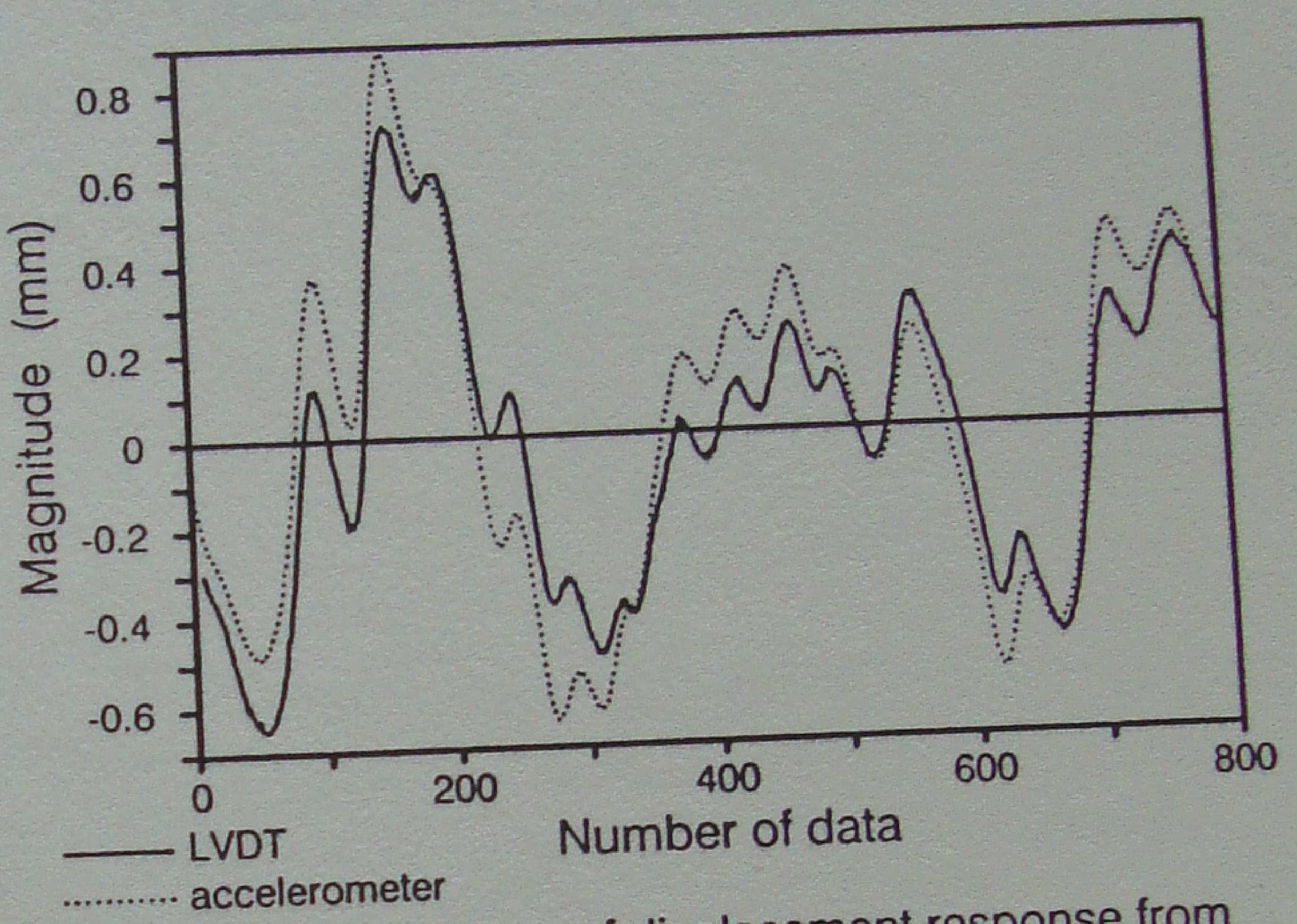


Figure 11: Comparison of displacement response from LVDT and accelerometer, random input